A CHARTOGRAPHY OF SPACETIME AROUND SMBHs with extreme-mass ratio inspirals

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Latin American Webinar on Physics, 2021

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High-energy phenomena

POWER OUTPUT OF WEAK SOURCES, SUCH AS GW150914

[Snapshot from press release 11 Feb 2016, https://youtu.be/vd1Pak5f6GQ]

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- This is $\sim 10^{61}\mathbf{ergs} \sim 5 \times 10^{60}\mathbf{TeV}$

A universe of black holes

BLACK HOLES COME IN DIFFERENT FLAVORS

*[Fig. 3 (trimmed) from first M87 Event Horizon Telescope Results.
I. The Shadow of the Supermassive Black Hole.
The Event Horizon Telescope Collaboration et al. 2019, ApJL 875 L1]*

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- **Intermediate-mass black holes:**
  Formation unclear, $100 \lesssim M_{\rm IMBH}/M_\odot < 10^5$, **probably in clusters**

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WAIT...

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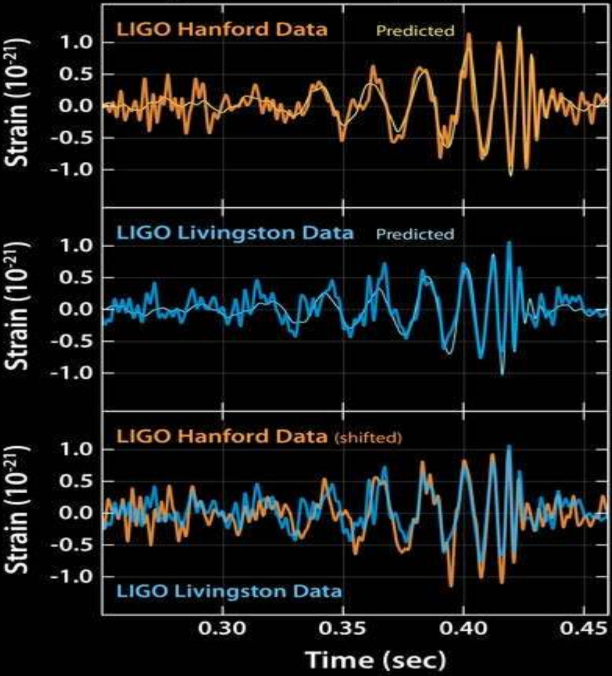
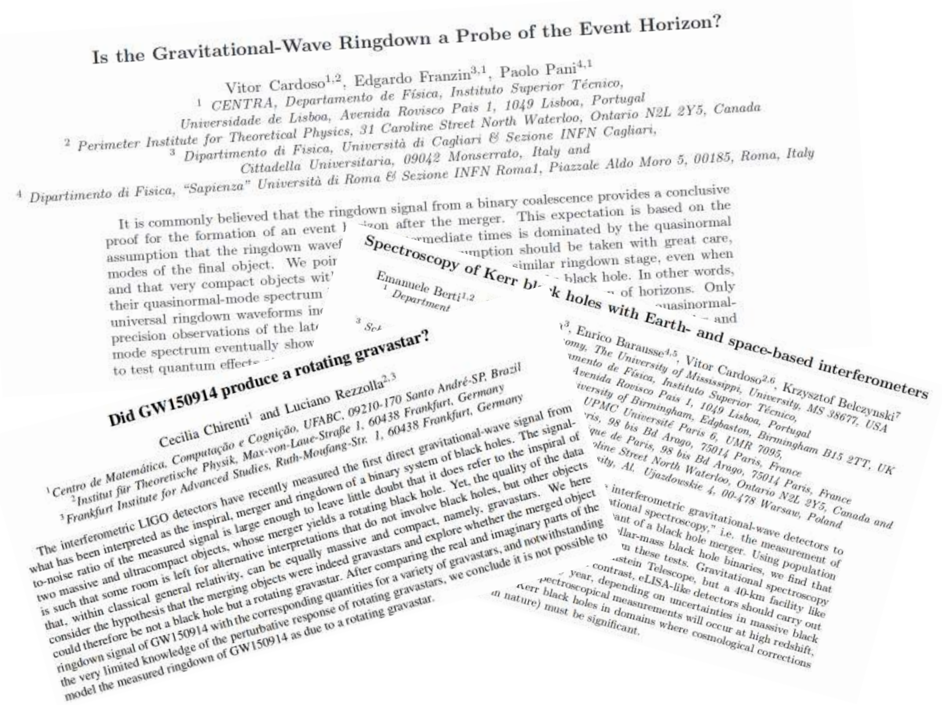
Image Credit: Caltech/MIT/LIGO Lab



So, these waves are a proof for the existence of black holes... right?



Is the Gravitational-Wave Ringdown a Probe of the Event Horizon?

Vitor Cardoso[1,2], Edgardo Franzin[3,1], Paolo Pani[4,1]
[1] *CENTRA, Departamento de Física, Instituto Superior Técnico,*
Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal
[2] *Perimeter Institute for Theoretical Physics, 31 Caroline Street North Waterloo, Ontario N2L 2Y5, Canada*
[3] *Dipartimento di Fisica, Università di Cagliari & Sezione INFN Cagliari,*
Cittadella Universitaria, 09042 Monserrato, Italy and
[4] *Dipartimento di Fisica, "Sapienza" Università di Roma & Sezione INFN Roma1, Piazzale Aldo Moro 5, 00185, Roma, Italy*

It is commonly believed that the ringdown signal from a binary coalescence provides a conclusive
proof for the formation of an event h izon after the merger. This expectation is based on the
assumption that the ringdown wavef rmediate times is dominated by the quasinormal
modes of the final object. We poi mption should be taken with great care,
and that very compact objects wit similar ringdown stage, even when
their quasinormal-mode spectrum black hole. In other words,
universal ringdown waveforms in of horizons. Only
precision observations of the lat quasinormal-
mode spectrum eventually show and
to test quantum effects

Spectroscopy of Kerr bl k holes with Earth- and space-based interferometers

Emanuele Berti[1,2] [3], Enrico Barausse[4,5], Vitor Cardoso[2,6], Krzysztof Belczynski[7]
[1] *Department omy, The University of Mississippi, University, MS 38677, USA*
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que de Paris, 98 bis Bd Arago, 75014 Paris, France
oline Street North Waterloo, Ontario N2L 2Y5, Canada and
[3] *Sc sity, Al. Ujazdowskie 4, 00-478 Warsaw, Poland*

interferometric gravitational-wave detectors to
ational spectroscopy," i.e. the measurement of
ant of a black hole merger. Using population
llar-mass black hole binaries, we find that
n these tests. Gravitational spectroscopy
stein Telescope, but a 40-km facility like
contrast, eLISA-like detectors should carry out
year, depending on uncertainties in massive black
pectroscopical measurements will occur at high redshift,
Kerr black holes in domains where cosmological corrections
n nature) must be significant.

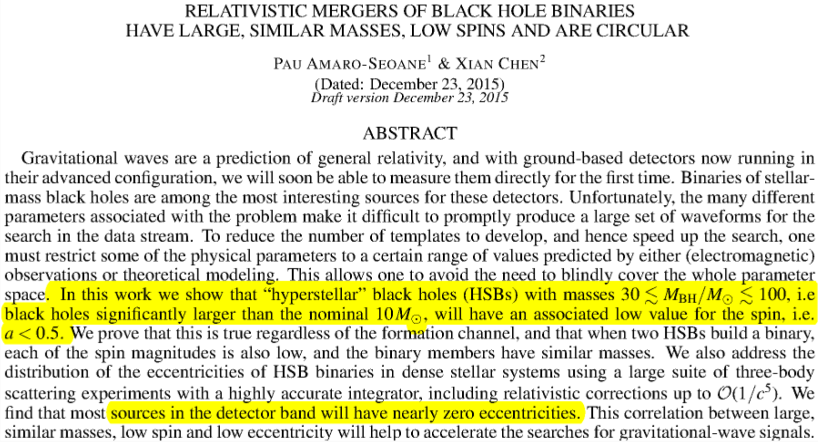
Did GW150914 produce a rotating gravastar?

Cecilia Chirenti[1] and Luciano Rezzolla[2,3]
[1] *Centro de Matemática, Computação e Cognição, UFABC, 09210-170 Santo André-SP, Brazil*
[2] *Institut für Theoretische Physik, Max-von-Laue-Straße 1, 60438 Frankfurt, Germany*
[3] *Frankfurt Institute for Advanced Studies, Ruth-Moufang-Str. 1, 60438 Frankfurt, Germany*

The interferometric LIGO detectors have recently measured the first direct gravitational-wave signal from what has been interpreted as the inspiral, merger and ringdown of a binary system of black holes. The signal-to-noise ratio of the measured signal is large enough to leave little doubt that it does refer to the inspiral of two massive and ultracompact objects, whose merger yields a rotating black hole. Yet, the quality of the data is such that some room is left for alternative interpretations that do not involve black holes, but other objects that, within classical general relativity, can be equally massive and compact, namely, gravastars. We here consider the hypothesis that the merging objects were indeed gravastars and explore whether the merged object could therefore be not a black hole but a rotating gravastar. After comparing the real and imaginary parts of the ringdown signal of GW150914 with the corresponding quantities for a variety of gravastars, and notwithstanding the very limited knowledge of the perturbative response of rotating gravastars, we conclude it is not possible to model the measured ringdown of GW150914 as due to a rotating gravastar.

A PREDICTION

BACK IN 2015



RELATIVISTIC MERGERS OF BLACK HOLE BINARIES HAVE LARGE, SIMILAR MASSES, LOW SPINS AND ARE CIRCULAR

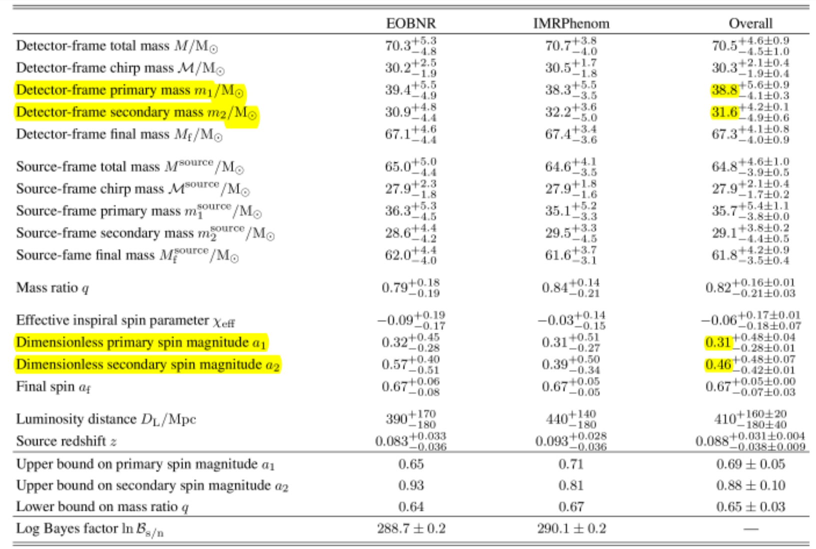
PAU AMARO-SEOANE[1] & XIAN CHEN[2]

(Dated: December 23, 2015)
Draft version December 23, 2015

ABSTRACT

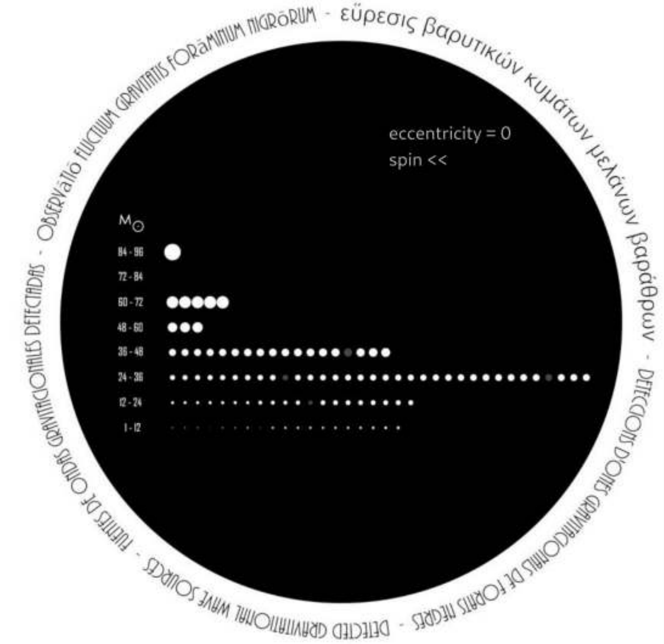
Gravitational waves are a prediction of general relativity, and with ground-based detectors now running in their advanced configuration, we will soon be able to measure them directly for the first time. Binaries of stellar-mass black holes are among the most interesting sources for these detectors. Unfortunately, the many different parameters associated with the problem make it difficult to promptly produce a large set of waveforms for the search in the data stream. To reduce the number of templates to develop, and hence speed up the search, one must restrict some of the physical parameters to a certain range of values predicted by either (electromagnetic) observations or theoretical modeling. This allows one to avoid the need to blindly cover the whole parameter space. In this work we show that "hyperstellar" black holes (HSBs) with masses $30 \lesssim M_{\rm BH}/M_{\odot} \lesssim 100$, i.e black holes significantly larger than the nominal $10\,M_{\odot}$, will have an associated low value for the spin, i.e. $a < 0.5$. We prove that this is true regardless of the formation channel, and that when two HSBs build a binary, each of the spin magnitudes is also low, and the binary members have similar masses. We also address the distribution of the eccentricities of HSB binaries in dense stellar systems using a large suite of three-body scattering experiments with a highly accurate integrator, including relativistic corrections up to $\mathcal{O}(1/c^5)$. We find that most sources in the detector band will have nearly zero eccentricities. This correlation between large, similar masses, low spin and low eccentricity will help to accelerate the searches for gravitational-wave signals.

February 2016: First detection presented



	EOBNR	IMRPhenom	Overall
Detector-frame total mass $M/\mathrm{M}_\odot$	$70.3^{+5.3}_{-4.8}$	$70.7^{+3.8}_{-4.0}$	$70.5^{+4.6\pm0.9}_{-4.5\pm1.0}$
Detector-frame chirp mass $\mathcal{M}/\mathrm{M}_\odot$	$30.2^{+2.5}_{-1.9}$	$30.5^{+1.7}_{-1.8}$	$30.3^{+2.1\pm0.4}_{-1.9\pm0.4}$
Detector-frame primary mass $m_1/\mathrm{M}_\odot$	$39.4^{+5.5}_{-4.9}$	$38.3^{+5.5}_{-3.5}$	$38.8^{+5.6\pm0.9}_{-4.1\pm0.3}$
Detector-frame secondary mass $m_2/\mathrm{M}_\odot$	$30.9^{+4.8}_{-4.4}$	$32.2^{+3.6}_{-5.0}$	$31.6^{+4.2\pm0.1}_{-4.9\pm0.6}$
Detector-frame final mass $M_\mathrm{f}/\mathrm{M}_\odot$	$67.1^{+4.6}_{-4.4}$	$67.4^{+3.4}_{-3.6}$	$67.3^{+4.1\pm0.8}_{-4.0\pm0.9}$
Source-frame total mass $M^\mathrm{source}/\mathrm{M}_\odot$	$65.0^{+5.0}_{-4.4}$	$64.6^{+4.1}_{-3.5}$	$64.8^{+4.6\pm1.0}_{-3.9\pm0.5}$
Source-frame chirp mass $\mathcal{M}^\mathrm{source}/\mathrm{M}_\odot$	$27.9^{+2.3}_{-1.8}$	$27.9^{+1.8}_{-1.6}$	$27.9^{+2.1\pm0.4}_{-1.7\pm0.2}$
Source-frame primary mass $m_1^\mathrm{source}/\mathrm{M}_\odot$	$36.3^{+5.3}_{-4.5}$	$35.1^{+5.2}_{-3.3}$	$35.7^{+5.4\pm1.1}_{-3.8\pm0.0}$
Source-frame secondary mass $m_2^\mathrm{source}/\mathrm{M}_\odot$	$28.6^{+4.4}_{-4.2}$	$29.5^{+3.3}_{-4.5}$	$29.1^{+3.8\pm0.2}_{-4.4\pm0.5}$
Source-fame final mass $M_\mathrm{f}^\mathrm{source}/\mathrm{M}_\odot$	$62.0^{+4.4}_{-4.0}$	$61.6^{+3.7}_{-3.1}$	$61.8^{+4.2\pm0.9}_{-3.5\pm0.4}$
Mass ratio q	$0.79^{+0.18}_{-0.19}$	$0.84^{+0.14}_{-0.21}$	$0.82^{+0.16\pm0.01}_{-0.21\pm0.03}$
Effective inspiral spin parameter χ_eff	$-0.09^{+0.19}_{-0.17}$	$-0.03^{+0.14}_{-0.15}$	$-0.06^{+0.17\pm0.01}_{-0.18\pm0.07}$
Dimensionless primary spin magnitude a_1	$0.32^{+0.45}_{-0.28}$	$0.31^{+0.51}_{-0.27}$	$0.31^{+0.48\pm0.04}_{-0.28\pm0.01}$
Dimensionless secondary spin magnitude a_2	$0.57^{+0.40}_{-0.51}$	$0.39^{+0.50}_{-0.34}$	$0.46^{+0.48\pm0.07}_{-0.42\pm0.01}$
Final spin a_f	$0.67^{+0.06}_{-0.08}$	$0.67^{+0.05}_{-0.05}$	$0.67^{+0.05\pm0.00}_{-0.07\pm0.03}$
Luminosity distance $D_\mathrm{L}/\mathrm{Mpc}$	390^{+170}_{-180}	440^{+140}_{-180}	$410^{+160\pm20}_{-180\pm40}$
Source redshift z	$0.083^{+0.033}_{-0.036}$	$0.093^{+0.028}_{-0.036}$	$0.088^{+0.031\pm0.004}_{-0.038\pm0.009}$
Upper bound on primary spin magnitude a_1	0.65	0.71	0.69 ± 0.05
Upper bound on secondary spin magnitude a_2	0.93	0.81	0.88 ± 0.10
Lower bound on mass ratio q	0.64	0.67	0.65 ± 0.03
Log Bayes factor $\ln \mathcal{B}_\mathrm{s/n}$	288.7 ± 0.2	290.1 ± 0.2	—

IT LOOKS WE GOT IT RIGHT



Why is this so important?

- **From the point of view of fundamental physics:**

1. LIGO/Virgo detections of mergers and the inspiral are a prediction of general relativity in the strong regime ... read “GR is correct”

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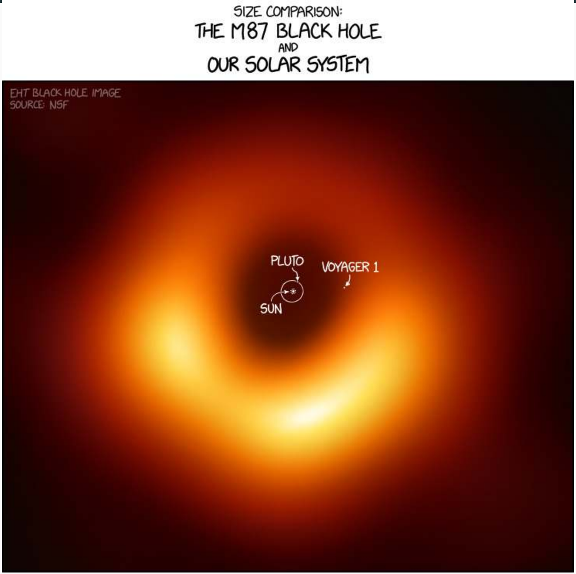
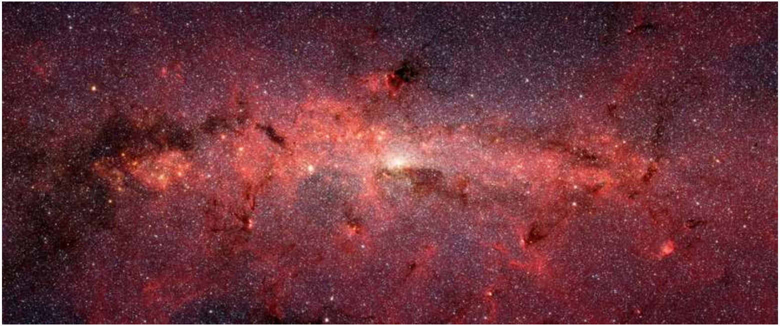
Do supermassive black holes exist?

It looks like that...



[From https://xkcd.com/2135/, using the figure from the EHT team web page, https://eventhorizontelescope.org/]

THE SMBH IN OUR GALAXY: OUR CLOSEST CANDIDATE



[NASA/JPL-Caltech/S. Stolovy (SSC/Caltech)]

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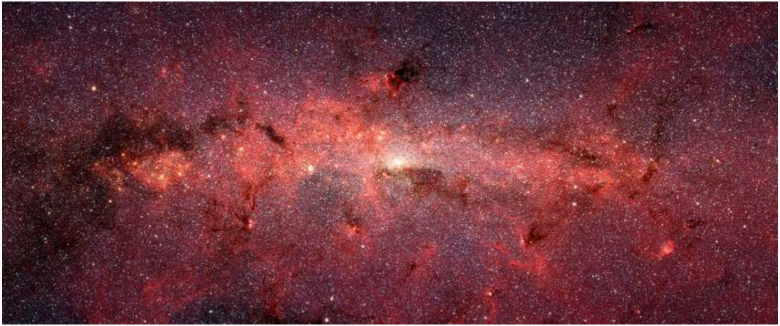

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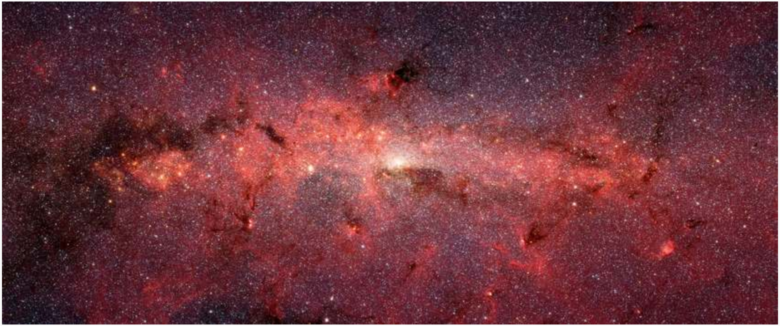

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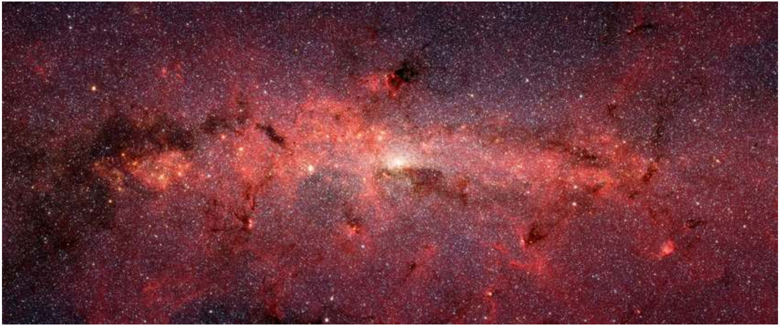

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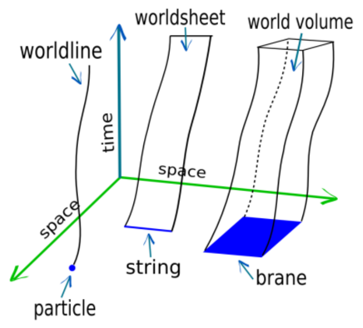
- Four millions of solar masses, **four millions of Suns**
- Within a radius of 22 millions of km, **enclosed in** $\sim 1/3$ **times the distance between the Earth and the Sun**

[Video: S-Stars, win+1]

Singularities: Why are they interesting?

WORLDLINES



[Credits: https://en.wikipedia.org/wiki/User:Stevertigo]

- A singularity is not a point in spacetime where gravity becomes infinite. **It is a much more subtle matter**

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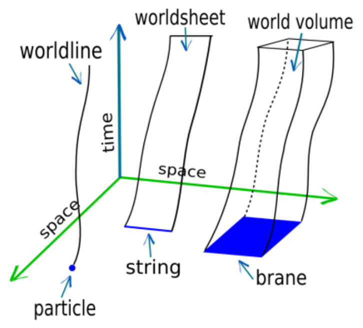

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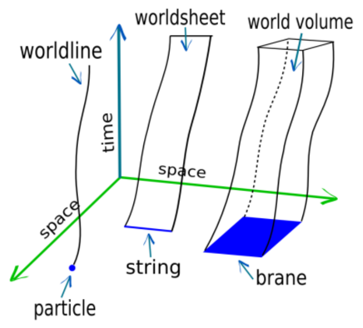

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- ... and admits a causal curve which has a finite past or future **(or both)**

EXAMPLES OF SINGULAR SPACETIMES

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  There is a finite past

HOW TO INTERPRET THEM?

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- What happens beyond that **is anyone's guess.**

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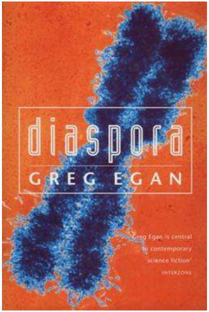
(3) Causality. **Spacetime does not have closed causal curves.**

CONNECTION BETWEEN GEOMETRY AND TOPOLOGY



[By Source, Fair use, https://en.wikipedia.org/w/index.php?curid=41668112]

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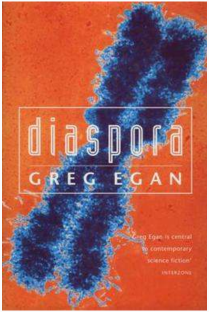

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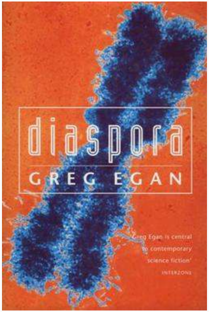

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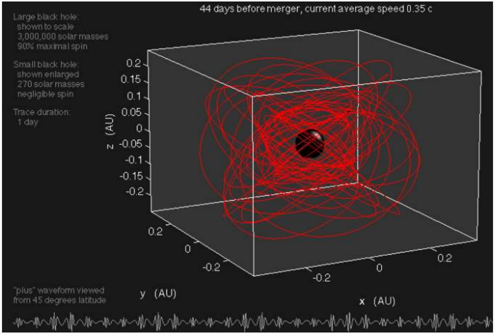
Extreme-Mass Ratio Inspirals:
Getting as close as we can

Extreme-Mass Ratio Inspirals

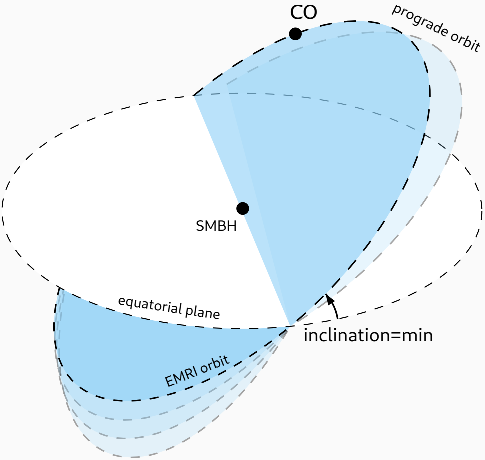


[Video: Extreme-mass ratio inspiral, by S. Drasco win+2 and Natalia win+3]

- Stellar-mass object spiraling into $10^4 - 10^6\ M_\odot$
- Such massive black holes are hosted in relaxed galactic nuclei (!)
- With LISA z $\sim 1, 4$

[Amaro-Seoane 2018, Babak et al +Amaro-Seoane 2017, Amaro-Seoane et al 2007]

AN ILLUSTRATION



[Amaro-Seoane 2021]

FUNDAMENTAL PHYSICS IN THE STRONG GRAVITY REGIME

❶ **Extreme-mass ratio inspirals**

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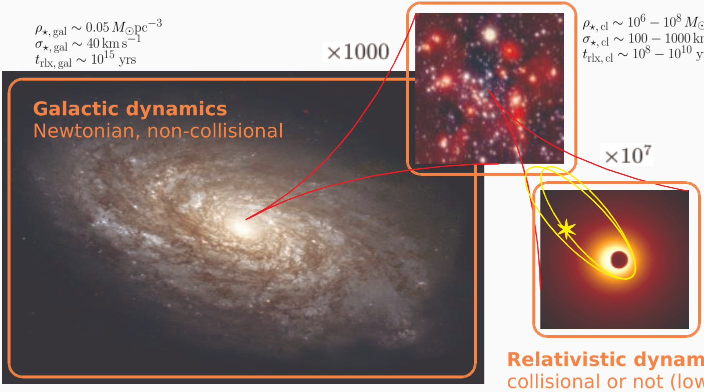
A PROBLEM OF 10 ORDERS OF MAGNITUDE



$\rho_{\star,\,\mathrm{gal}} \sim 0.05\,M_\odot \mathrm{pc}^{-3}$
$\sigma_{\star,\,\mathrm{gal}} \sim 40\,\mathrm{km\,s^{-1}}$
$t_{\mathrm{rlx,\,gal}} \sim 10^{15}$ yrs

Galactic dynamics
Newtonian, non-collisional

$\times 1000$

Cluster dynamics
Newtonian, collisional

$\rho_{\star,\,\mathrm{cl}} \sim 10^6 - 10^8\,M_\odot\,\mathrm{pc}^{-3}$
$\sigma_{\star,\,\mathrm{cl}} \sim 100 - 1000\,\mathrm{km\,s^{-1}}$
$t_{\mathrm{rlx,\,cl}} \sim 10^8 - 10^{10}$ yrs

$\times 10^7$

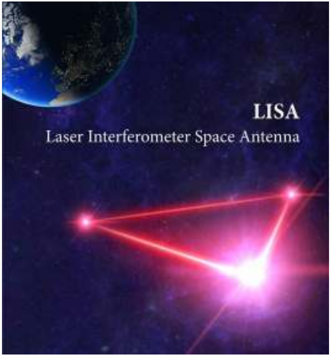
Relativistic dynamics
collisional or not (low *N*)
$\mathcal{M}_\bullet \sim 10^6 - 10^9\,M_\odot$
$R_{\mathrm{Schw}} = 10^{-7} - 10^{-4}$ pc

Note: $1pc \sim 3$ **light years**

[Amaro-Seoane 2012]

Can we detect them?

THE LASER INTERFEROMETER SPACE ANTENNA



[Trimmed from original figure by NASA/Simon Barke]

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[Amaro-Seoane et al 2017, arXiv170200786A]

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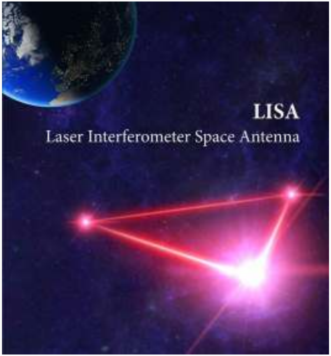

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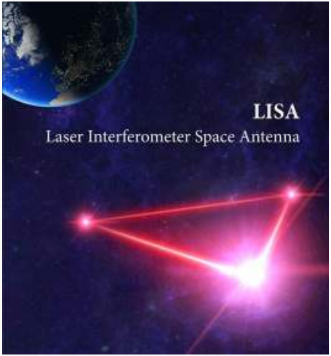

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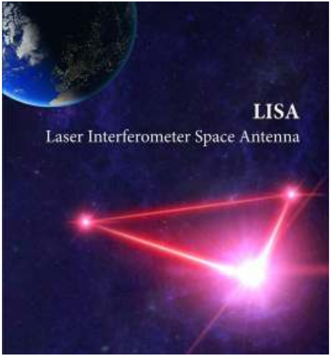

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- On June 20, 2017 **LISA received its clearance goal from ESA**

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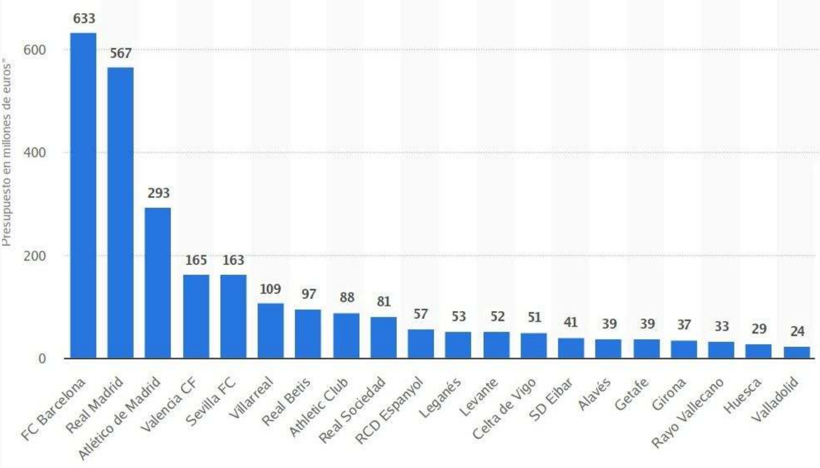
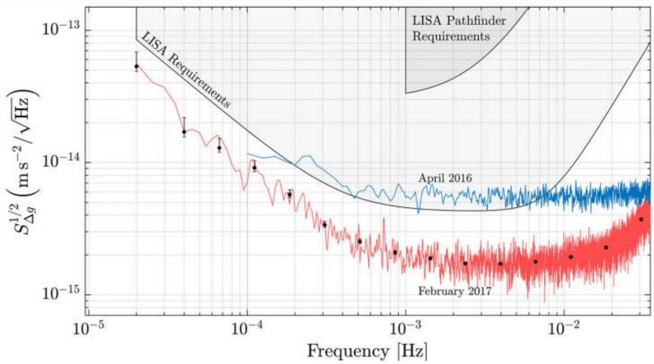
LISA: ABOUT 1/2 OF THE COST 1ST DIVISION SPANISH LIGA (2018/2019)



[Source: https://es.statista.com/estadisticas/498947/presupuesto-equipos-de-futbol-de-la-liga-en-espana/]

LISA PATHFINDER - A TRUE ACHIEVEMENT



[Armano et al. 2016, Fig. 1]

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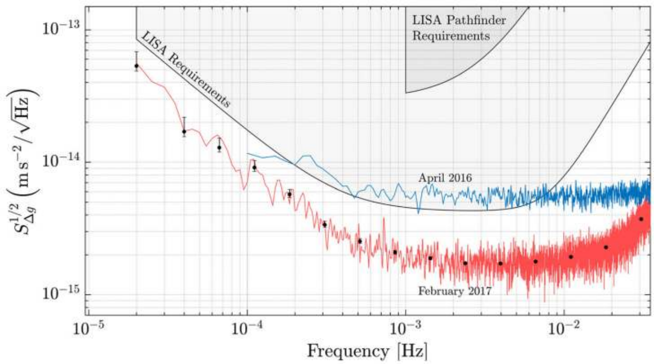

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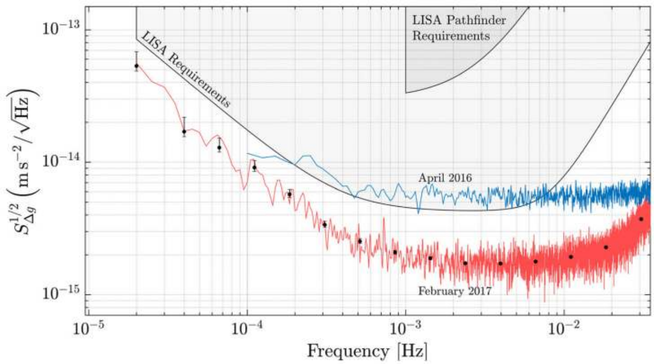

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- Sub-Femto-g Free Fall, **and differential acceleration measurements at 1 mHz can be done**
- Approached and overtook **LISA design requirements**

Do we really need to wait 15(-ish) years to detect captures?

“NO”

IMRIs

- **“Detecting intermediate-mass ratio inspirals from the ground and space”**

  Pau Amaro Seoane, Physical Review D, Volume 98, Issue 6, 2018

- **“Relativistic dynamics and extreme mass ratio inspirals”**

  Pau Amaro Seoane, Living Reviews in Relativity, Volume 21, Issue 1, article id. 4, 150 pp. 2018

- **“Investigating the retention of intermediate-mass black holes in star clusters using N-body simulations”**

  Konstantinidis, S.; Amaro-Seoane, P.; Kokkotas, K. D., Astronomy & Astrophysics, Volume 557, id.A135, 8 pp., 2013

- **“Laser Interferometer Space Antenna”**

  Pau Amaro Seoane et al., ESA call for missions for the L3 slot in the Cosmic Vision Programme

IMRIs CAN BE DETECTED RIGHT NOW

- Intermediate-mass ratio inspirals

[Konstantinidis, Amaro-Seoane & Kokkotas 2013, Amaro-Seoane 2018]

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  have lower masses and fall in the ET and/or LIGO/Virgo band

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[Konstantinidis, Amaro-Seoane & Kokkotas 2013, Amaro-Seoane 2018]

IMBHs

[IMBH in NGC 3783, Credit: ESO/M. Kornmesser]

- **We know that supermassive black holes correlate with the host galaxy:**

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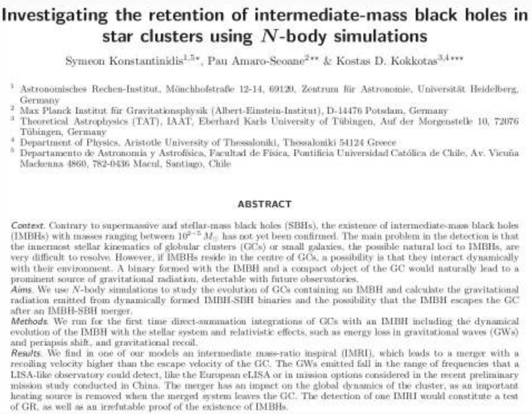
First numerical simulations of IMRIs

First numerical simulation of an IMRI



Investigating the retention of intermediate-mass black holes in star clusters using N-body simulations

Symeon Konstantinidis[1,5*], Pau Amaro-Seoane[2**] & Kostas D. Kokkotas[3,4***]

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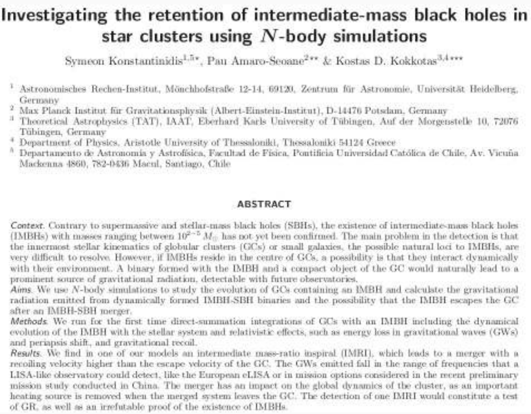
ABSTRACT

Context. Contrary to supermassive and stellar-mass black holes (SBHs), the existence of intermediate-mass black holes (IMBHs) with masses ranging between $10^{2-5}\,M_{\odot}$ has not yet been confirmed. The main problem in the detection is that the innermost stellar kinematics of globular clusters (GCs) or small galaxies, the possible natural loci to IMBHs, are very difficult to resolve. However, if IMBHs reside in the centre of GCs, a possibility is that they interact dynamically with their environment. A binary formed with the IMBH and a compact object of the GC would naturally lead to a prominent source of gravitational radiation, detectable with future observatories.
Aims. We use N-body simulations to study the evolution of GCs containing an IMBH and calculate the gravitational radiation emitted from dynamically formed IMBH-SBH binaries and the possibility that the IMBH escapes the GC after an IMBH-SBH merger.
Methods. We run for the first time direct-summation integrations of GCs with an IMBH including the dynamical evolution of the IMBH with the stellar system and relativistic effects, such as energy loss in gravitational waves (GWs) and periapsis shift, and gravitational recoil.
Results. We find in one of our models an intermediate mass-ratio inspiral (IMRI), which leads to a merger with a recoiling velocity higher than the escape velocity of the GC. The GWs emitted fall in the range of frequencies that a LISA-like observatory could detect, like the European eLISA or in mission options considered in the recent preliminary mission study conducted in China. The merger has an impact on the global dynamics of the cluster, as an important heating source is removed when the merged system leaves the GC. The detection of one IMRI would constitute a test of GR, as well as an irrefutable proof of the existence of IMBHs.

- Direct-summation code **written from scratch for special-purpose hardware: Myriad**

First numerical simulation of an IMRI



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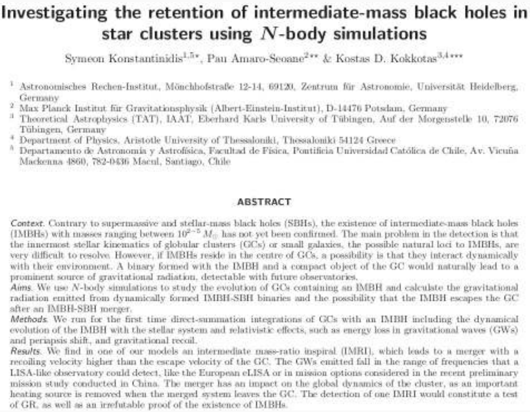

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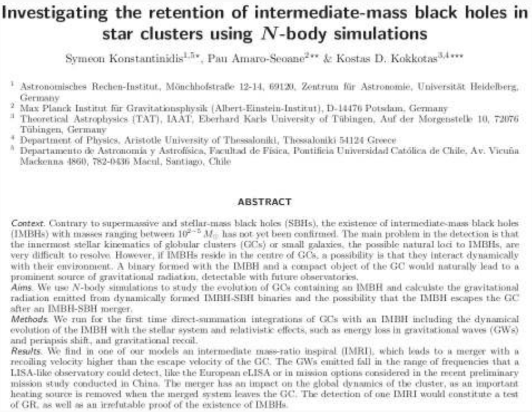
Aims. We use N-body simulations to study the evolution of GCs containing an IMBH and calculate the gravitational radiation emitted from dynamically formed IMBH-SBH binaries and the possibility that the IMBH escapes the GC after an IMBH-SBH merger.
Methods. We run for the first time direct-summation integrations of GCs with an IMBH including the dynamical evolution of the IMBH with the stellar system and relativistic effects, such as energy loss in gravitational waves (GWs) and periapsis shift, and gravitational recoil.
Results. We find in one of our models an intermediate mass-ratio inspiral (IMRI), which leads to a merger with a recoiling velocity higher than the escape velocity of the GC. The GWs emitted fall in the range of frequencies that a LISA-like observatory could detect, like the European eLISA or in mission options considered in the recent preliminary mission study conducted in China. The merger has an impact on the global dynamics of the cluster, as an important heating source is removed when the merged system leaves the GC. The detection of one IMRI would constitute a test of GR, as well as an irrefutable proof of the existence of IMBHs.

- Direct-summation code **written from scratch for special-purpose hardware: Myriad**
- Aim: Simulate stellar clusters realistically **including massive black holes**
- All phenomena included at least at lowest order

FIRST NUMERICAL SIMULATION OF AN IMRI



Investigating the retention of intermediate-mass black holes in star clusters using N-body simulations

Symeon Konstantinidis[1,5]*, Pau Amaro-Seoane[2]** & Kostas D. Kokkotas[3,4]***

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[3] Theoretical Astrophysics (TAT), IAAT, Eberhard Karls University of Tübingen, Auf der Morgenstelle 10, 72076 Tübingen, Germany
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[5] Departamento de Astronomía y Astrofísica, Facultad de Física, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, 782-0436 Macul, Santiago, Chile

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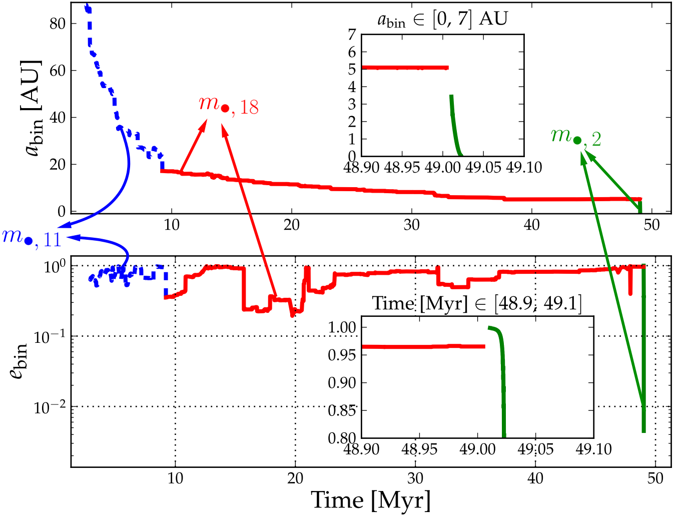
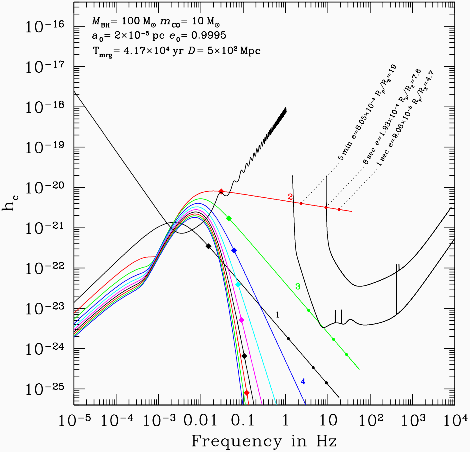
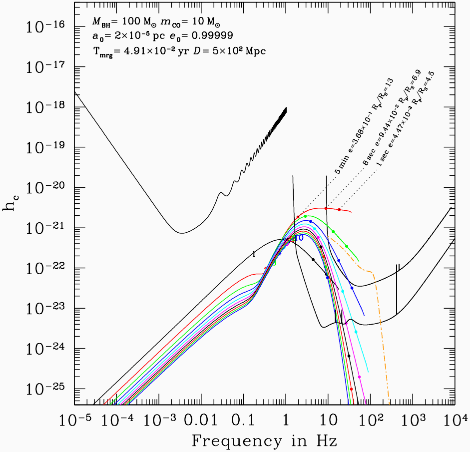
- Aim: Simulate stellar clusters realistically **including massive black holes**
- All phenomena included at least at lowest order
  In particular: relativistic corrections and recoil

DYNAMICAL EVOLUTION

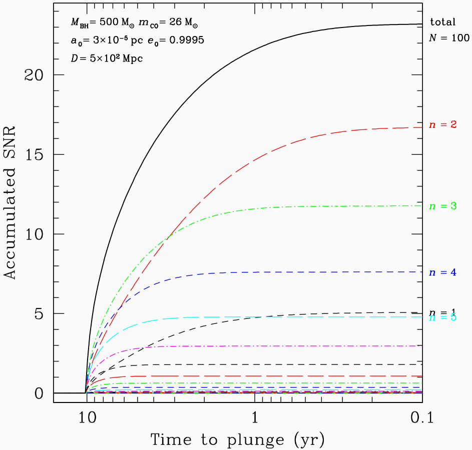


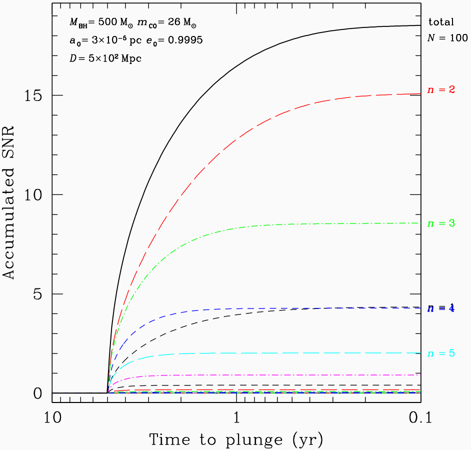
YEARS OF FOREWARN WITH SECONDS OF PRECISION



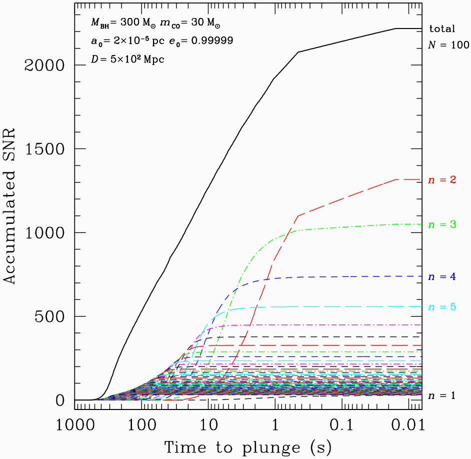


LISA, 10 AND 5 YRS BEFORE MERGER

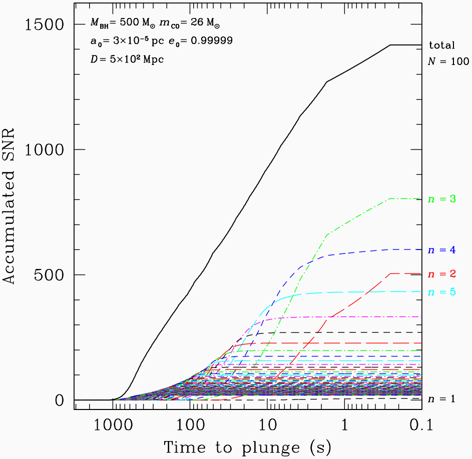




ET

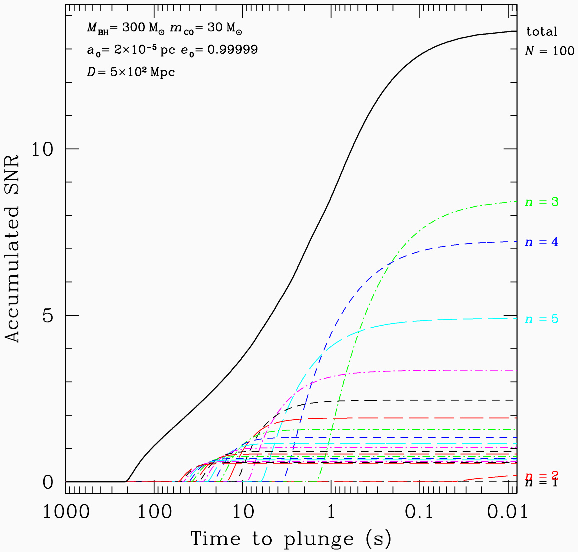


M_{BH} = 300 $M_\odot$ m_{CO} = 30 $M_\odot$
a_0 = 2×10^{-5} pc e_0 = 0.99999
D = 5×10^2 Mpc
total
N = 100
n = 2
n = 3
n = 4
n = 5
n = 1
Accumulated SNR
0
500
1000
1500
2000
1000
100
10
1
0.1
0.01
Time to plunge (s)



M_{BH} = 500 $M_\odot$ m_{CO} = 26 $M_\odot$
a_0 = 3×10^{-5} pc e_0 = 0.99999
D = 5×10^2 Mpc
total
N = 100
n = 3
n = 4
n = 2
n = 5
n = 1
Accumulated SNR
0
500
1000
1500
1000
100
10
1
0.1
Time to plunge (s)

LIGO/VIRGO



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D = 5×10^2 Mpc
Accumulated SNR
0
5
10
Time to plunge (s)
1000
100
10
1
0.1
0.01
total
N = 100
n = 3
n = 4
n = 5
n = 2
n = 1

SPIN MATTERS

- **"THE ROLE OF THE SMBH SPIN IN THE ESTIMATION OF THE EMRI EVENT RATE"**

  PAU AMARO SEOANE ET AL., MNRAS VOLUME 429, ISSUE 4, 2013

- **"RELATIVISTIC DYNAMICS AND EXTREME MASS RATIO INSPIRALS"**

  PAU AMARO SEOANE, LIVING REVIEWS IN RELATIVITY, VOLUME 21, ISSUE 1, ARTICLE ID. 4, 150 PP. 2018

- **"LASER INTERFEROMETER SPACE ANTENNA"**

  PAU AMARO SEOANE ET AL., ESA CALL FOR MISSIONS FOR THE L3 SLOT IN THE COSMIC VISION PROGRAMME

PLUNGES

- Perturbations at pericenter from bulk of system are dangerous

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  Deflect it off or make it plunge through horizon

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  But number density drops orders of magnitude

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  But number density drops orders of magnitude
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  Plunges are more frequent than adiabatic EMRIs

NOT AN ACADEMIC EXERCISE

- Number of periapsis passages for an extremely radial EMRI

NOT AN ACADEMIC EXERCISE

- Number of periapsis passages for an extremely radial EMRI **before it plunges?**

NOT AN ACADEMIC EXERCISE

- Number of periapsis passages for an extremely radial EMRI
  before it plunges?
- Calculate (E, L_z, C) and their average time evolution

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  Derive number of periapsis passages

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- Depending on ι and s...

NOT AN ACADEMIC EXERCISE

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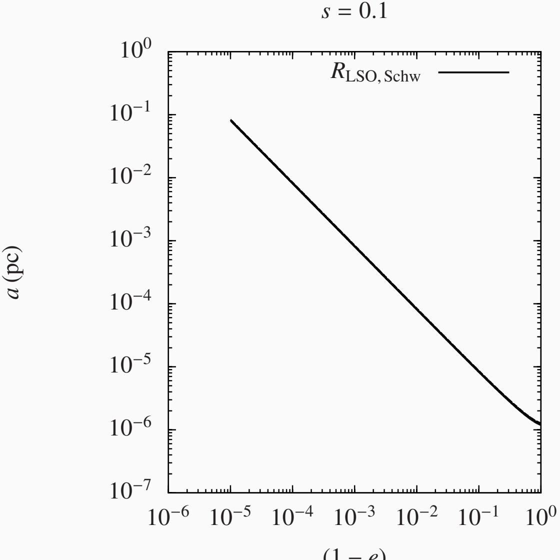
  before it plunges?
- Calculate (E, L_z, C) and their average time evolution

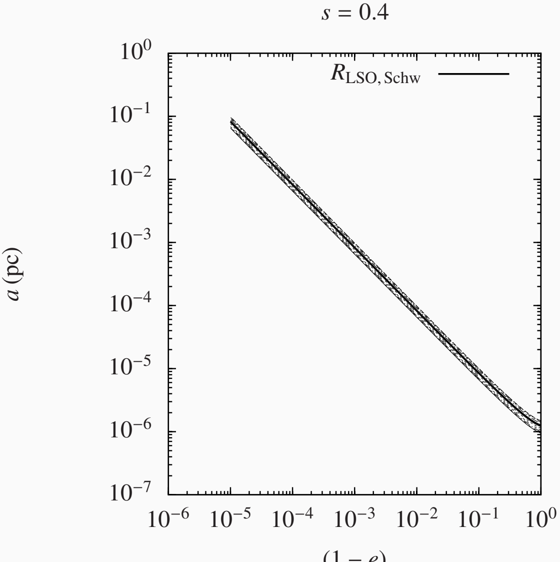
  Derive number of periapsis passages
- Depending on ι and s...

  ... 10^3 **and** 10^5 **passages in the bandwidth**

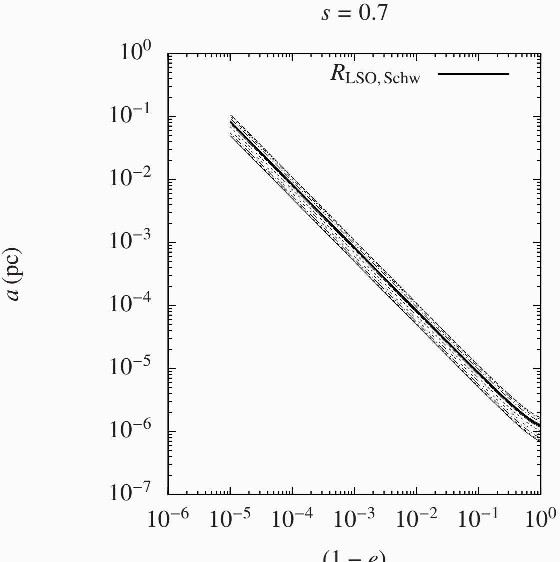
A FAMILY OF SEPARATRICES: $s = 0.1$



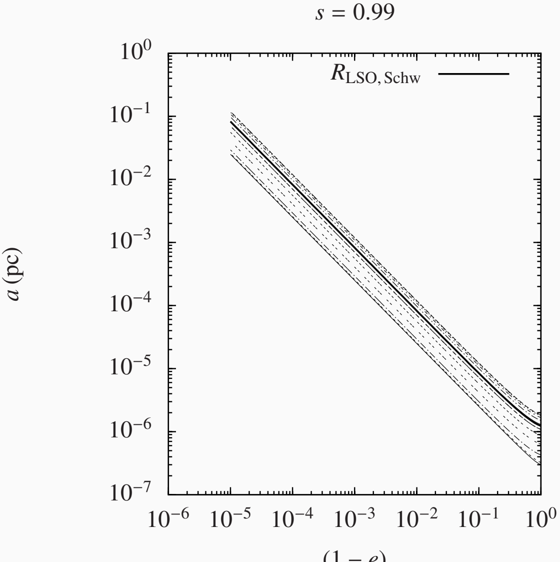
A FAMILY OF SEPARATRICES: $s = 0.4$



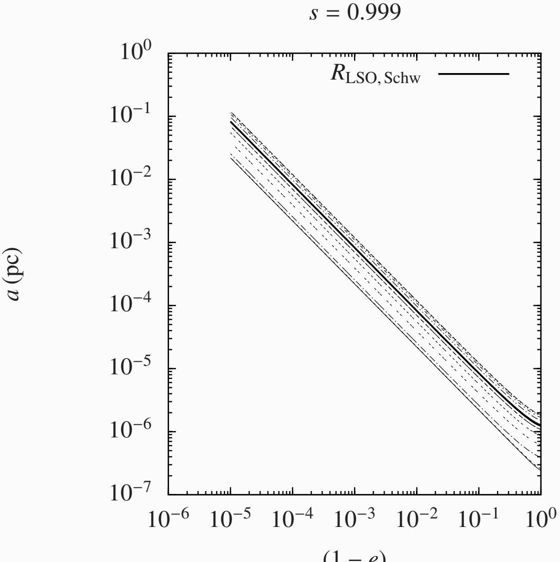
A FAMILY OF SEPARATRICES: $s = 0.7$



A FAMILY OF SEPARATRICES: $s = 0.99$

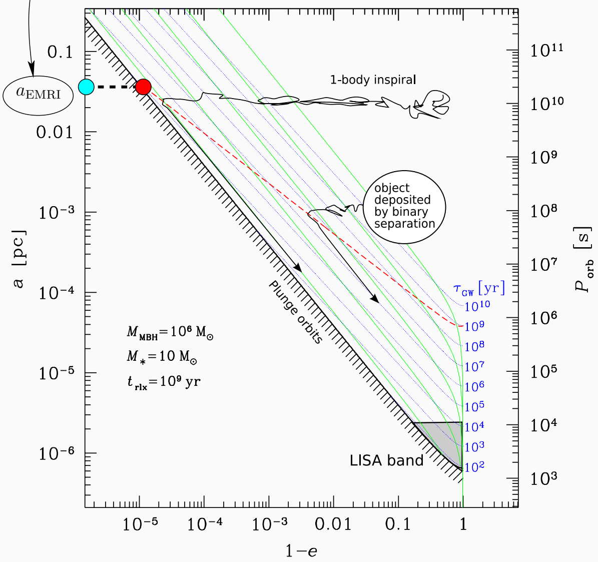


A FAMILY OF SEPARATRICES: $s = 0.999$



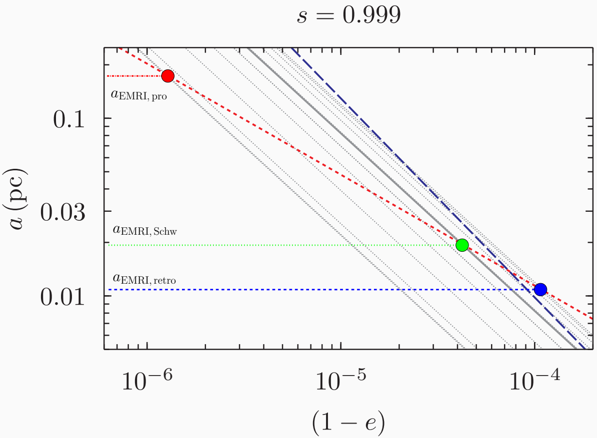
IMPACT OF THE SPIN ON THE RATES?

$$\dot{N}_{\rm EMRI} \simeq \int_0^{a_{\rm EMRI}} \frac{dN_\bullet(a)}{\ln\left(\theta_{\rm LC}^{-2}\right) t_r(a)}$$



IT'S ALL ABOUT AN UPPER LIMIT

$$\dot{N}_{\rm EMRI} \simeq \int_0^{a_{\rm EMRI}} \frac{dN_\bullet(a)}{\ln\left(\theta_{\rm LC}^{-2}\right)\, t_r(a)}$$



KERR VS. SCHWARZSCHILD

$$a_{\mathrm{EMRI}}^{\mathrm{Kerr}} = a_{\mathrm{EMRI}}^{\mathrm{Schw}} \times \mathcal{W}^{\frac{-5}{6-2\gamma}}(\iota,\, s)$$

$$\dot{N}_{\mathrm{EMRI}}^{\mathrm{Kerr}} = \dot{N}_{\mathrm{EMRI}}^{\mathrm{Schw}} \times \mathcal{W}^{\frac{20\gamma-45}{12-4\gamma}}(\iota,\, s)$$

- Take a typical value of a prograde orbit with high spin: $\mathcal{W} = 0.15$; **then for a modest** $\gamma = 1.5$

$$\dot{N}_{\mathrm{EMRI}}^{\mathrm{Kerr}} \sim 114 \times \dot{N}_{\mathrm{EMRI}}^{\mathrm{Schw}}$$

Gravitational Wave and (candidate) black holes...

A check mark list

CHECK MARK LIST

- **Have LIGO and Virgo directly detect GWs ... ✓**

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- **Detect merging black holes ...** ✓

Check mark list

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CHECK MARK LIST

- **Have LIGO and Virgo directly detect GWs ... ✓**
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- **... and the associated electromagnetic counterpart ... ✓**

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- **Open up multimessenger astronomy ...** ✓

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- **Get LISA have a firm launch slot at ESA ... ✓**

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- **“See” the event horizon of M87 thanks to the EHT ... ✓**

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- **Get LISA have a firm launch slot at ESA ... ✓**
- **Get the 2017 Nobel Prize ... ✓**
- **"See" the event horizon of M87 thanks to the EHT ... ✓**
- **Get the 2020 Nobel Prize ... ✓**
- **Get fashion involved in GWs ... ✓**

GW JACKETS

Seen at the Gangnam Station Underground Shopping Center, next to line 2, Seoul, 23/Dec/2017

A chartography of spacetime around SMBHs with extreme-mass ratio inspirals

Pau Amaro Seoane

amaro@riseup.net

Extra material

HARMONICS

The strain amplitude in the n-th harmonic at a given distance D, normalized to the typical values of this work is

$$\begin{aligned} h_n &= g(n, e)\frac{G^2\, M_{\rm BH} m_{\rm CO}}{D\, a\, c^4} \\ &\simeq 8\times 10^{-23} g(n, e)\left(\frac{D}{500\,{\rm Mpc}}\right)^{-1}\left(\frac{a}{10^{-5}\,{\rm pc}}\right)^{-1} \\ &\left(\frac{M_{\rm BH}}{10^3\,M_\odot}\right)\left(\frac{m_{\rm CO}}{10\,M_\odot}\right). \end{aligned}$$

In this expression $M_{\rm BH}$ is the mass of the IMBH, $m_{\rm CO}$ is the mass of the compact object (CO), and $g(n,\ e)$ is a function of the harmonic number n and the eccentricity e *[Peters & Matthews 1963]*. We consider the RMS amplitude averaged over the two GW polarizations and all directions.

COMBINING DYNAMICS AND GRAVITATIONAL RADIATION

$$\dot{a}_{\mathrm{GW}} = -\frac{64}{5}\frac{G^3 M_{\mathrm{BH}}\, m_{\mathrm{CO}}(M_{\mathrm{BH}} + m_{\mathrm{CO}})}{c^5 a^3 (1-e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \tag{1}$$

$$\dot{e}_{\mathrm{GW}} = -\frac{304}{15}\frac{G^3 M_{\mathrm{BH}}\, m_{\mathrm{CO}}(M_{\mathrm{BH}} + m_{\mathrm{CO}})}{c^5 a^4 (1-e^2)^{5/2}} e\left(1 + \frac{121}{304}e^2\right) \tag{2}$$

The GW terms are as given in *[Peters 1964]*.

COMBINING DYNAMICS AND GRAVITATIONAL RADIATION

Using the relationships of *[Quinlan 1996]*, we have that

$$\dot{a}_{\mathrm{D}} = -H \frac{G\rho}{\sigma} a^2. \tag{3}$$

For the kind of binaries I am considering in this work, i.e. hard ones, we have that $(de/d\ln(1/a))_{\mathrm{D}} = K(e)$. Since the density drops significantly during the evolution, we can regard σ as approximately constant and hence $de = K(e)\, d\ln(1/a) = -K(e)/a\, da$, so that $H \simeq 16$, as in the original work of *[Quinlan 1996]*. Therefore,

$$\dot{e}_{\mathrm{D}} = \frac{H}{\sigma}\, G\rho\, a\, K(e), \tag{4}$$

with $K(e) \sim K_0\, e(1-e^2)$, as in the work of *[Merritt & Milosavljević 2005]*.

PHASE SHIFT

We can estimate the accumulated phase shift to lowest post-Newtonian order and to first order in e^2 with *[Krolak et al 1995]*

$$\Delta\Psi_e(f) = \Psi_{\rm last} - \Psi_{\rm i} \cong -\Psi_{\rm i} = \frac{7065}{187136}\, e_i^2\, (\pi\, f\, M_{\rm z})^{-5/3}\,. \tag{5}$$

f is the frequency for the $n = 2$ harmonic, and I have introduced the quantity $M_{\rm z} := (1+z) G\, (M_{\rm BH} \times m_{\rm CO})^{3/5}\, (M_{\rm BH} + m_{\rm CO})^{-1/5} / c^3$. Also, I make the approximation that $\Delta\Psi_{\rm e}(f) = \Psi_{\rm last} - \Psi_{\rm i} \simeq -\Psi_{\rm i}$, with $\Psi_{\rm last}$ and $\Psi_{\rm i}$ the final and initial phase. This is so because of the pronounced fall-off of $\Psi_{\rm e}(f)$ with increasing frequency, see discussion in section B.2 of *Cutler and Harms 2006*.

PHASE SHIFT

The semi-major axis of the binary is *[Kepler 1619]*

$$a^3 = \frac{G\,(M_{\rm BH} + m_{\rm CO})}{(\pi\,f)^2}. \tag{6}$$

The time for merger for $e \ll 1$ can be derived from *Peters 1964* as follows,

$$T_{\rm mrg} \cong \frac{5}{256}\,\frac{c^5}{G^3 M_{\rm BH} \times m_{\rm CO}\,(M_{\rm BH} + m_{\rm CO})} \left[\frac{G(M_{\rm BH} + m_{\rm CO})}{(\pi\,f)^2}\right]^{4/3}. \tag{7}$$

Last, let us recall that

$$e^2\,f^{19/9} \cong \text{constant}, \tag{8}$$

PHASE SHIFT

Therefore, if we use Eq. (**??**) in Eq. (**??**), we obtain

$$\pi f \cong \left(\frac{5}{256}\right)^{3/8} M_{\mathrm{z}}^{-5/8} T_{\mathrm{mrg}}^{-3/8}. \tag{9}$$

Hence, using Eqs. (**??**, **??**, **??**), we have that the accumulated phase shift in terms of f, $e_i(f)$, M_{z} and T_{mrg} is

$$\begin{aligned}\Delta\Psi_e(f) = \left(\frac{5}{256}\right)^{-17/12} \frac{7065}{187136} \\ (\pi f_i)^{19/9}\, e_i^2 M_{\mathrm{z}}^{25/36} T_{\mathrm{mrg}}^{17/12} \\ \cong 10\,(\pi f_i)^{19/9}\, e_i^2 M_{\mathrm{z}}^{25/36} T_{\mathrm{mrg}}^{17/12}\end{aligned} \tag{10}$$